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INTRODUCTION TO NUCLEAR PROPULSION.

Lecture 21: NUCLEAR ROCKET SYSTEM

Russell E. Motsinger

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INTRODUCTION TO NUCLEAR PROPULSION

Lecture 21 - NUCLEAR ROCKET SYSTEM

Russell E. Motsinger

May 14, 15, and 16, 1963

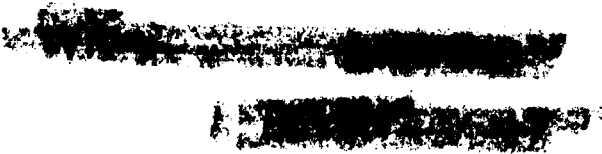
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[REDACTED]

PREFACE

Lecture 21 has been divided into Parts A and B. Otherwise the two parts are a unit and Part B requires Part A in order to maintain proper continuity.



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1. INTRODUCTION

This last lecture of the present series is concerned with some aspects of the nuclear rocket system as a whole in terms of what demands are likely to be made of it, how the addition of the need to provide shielding for manned capsules aboard the vehicle is likely to affect the minimum weight of the system, and how well such requirements are compatible with nuclear reactor capability.

Considerable attention is paid to the requirements for interplanetary travel in order to establish a hard-base definition of the minimum capability a rocket vehicle must have to demonstrate any real usefulness and, perhaps, to define the areas our ambitions can reasonably lead us by means of the addition of nuclear energy sources. The treatment is elementary insofar as mission analysis is concerned, which is proper for the present purpose of establishing a common base to which more rigid requirements can be added as the nature of the problem becomes more specific; albeit at the expense of freedom of choice when and how such missions can be performed, or with the danger of optimism on capability to perform certain marginal missions, the principal advantage gained by this approach is that of more general understanding. Those of you who have already worked in this area are aware of the extreme care required in defining reference systems for which energy balances can legitimately be made. A few comments are made on optimum mass ratio considering energy expenditure and on propulsive efficiency (a term of no practical utility for rockets).

Weight fractions for various nuclear rocket components are then discussed to set the basis for evaluation of requirements versus capability. The material is given in as general terms as possible, but certain choices are obviously required, hence are made to continue the discussion in practical terms. The implication of adding shadow shielding sufficient to protect man from reactor radiation in a crew compartment in front of the propellant tank is discussed.

Finally, some approximate but reasonably accurate tools-of-the-trade are included and used to assess the affect of postulated requirements likely to be expected from nuclear rockets. The assessment is not carried to a final conclusion, but sufficient procedure is given to establish which items are, and which are not important in the development of a configuration. Sufficient references are included for those interested to refine the procedure as much as desired.

2. ENERGY REQUIREMENTS

To appreciate the magnitude of the problem facing the rocket engineer from the viewpoint of performance, a review of the energy requirements, eventually translatable to reactor power density requirements, is in order. The power density is the final measure of the degree of difficulty to be anticipated in any nuclear reactor design.

2.1 VEHICLE ENERGY REQUIREMENT

The energy concept* is based upon the standard definitions of kinetic and potential energy:

$$\text{Kinetic energy} \quad E_k = \frac{MV^2}{2} \quad (2.1)$$

$$\text{Potential energy} \quad E_p = - \frac{\gamma MM_p}{r} \quad (2.2)$$

* Corliss, W. R., "Propulsion Systems for Space Flight", McGraw-Hill, 1960.

$$\begin{aligned} \text{Total energy } E_t &= E_k + E_p \\ &= \frac{MV^2}{2} - \frac{\gamma M M_p}{r} \end{aligned} \quad (2.3)$$

$$\text{or } \frac{E_t}{M} = \frac{V^2}{2} - \frac{\gamma M_p}{r} \quad (2.4)$$

The reference level chosen for potential energy measurements at $r = \infty$ so that $E_p = 0$ when the two bodies are infinitely distant from each other; the coordinate system is always centered at the principal gravitating body. The total energy given by this equation is that possessed by a body of mass, M , in an orbit of radius, r , at a tangential velocity, V , about the planetary body of mass, M_p . If the orbit is circular so that the centrifugal force balances the gravitational force,

$$\frac{MV_o^2}{r} = \frac{\gamma M M_p}{r^2} \quad (2.5)$$

$$\text{So: } V_o = \sqrt{\frac{\gamma M_p}{r}} \quad (2.6)$$

$$\text{and } \frac{E_t}{M} = \frac{\gamma M_p}{2r} - \frac{\gamma M_p}{r} = -\frac{\gamma M_p}{2r} \quad (2.7)$$

Thus the energy requirement per unit mass for a body in the gravity field is found for any radius, r . Interesting special cases consist of:

a) Escape velocity is reached when $E_k = E_p$

$$\frac{1}{2} M V_e^2 = \frac{\gamma M M_p}{r} \quad (2.8)$$

$$V_e = \sqrt{\frac{2\gamma M_p}{r}} = \sqrt{2} V_o \quad (2.9)$$

This also shows that the initial velocity, V_e , required to escape gravitational attraction is greater than the orbiting velocity, V_o , at the radius, r , by the factor $\sqrt{2}$.

* $\gamma = 6.67 \times 10^{-11} \frac{\text{Newtons } m^2}{kg^2}$; Newton-meter = 1 joule

b) The energy requirement for launch from surface to orbit is:

$$E_{K_1} = 0, \quad E_{K_2} = \frac{\gamma M M_p}{2r}$$

$$E_{p_1} = -\frac{\gamma M M_p}{R}, \quad E_{p_2} = -\frac{\gamma M M_p}{r}$$

$$= -MV_{oR}^2 \quad = -MV_{or}^2$$

$$\begin{aligned} \text{So } \Delta E_m &= [E_{K_2} + E_{p_2}] - [E_{K_1} + E_{p_1}] \\ &= \left[\frac{\gamma M M_p}{2r} - \frac{\gamma M M_p}{r} \right] - \left[0 - \frac{\gamma M M_p}{R} \right] \\ &= \frac{\gamma M M_p}{R} \left[1 - \frac{1}{2n} \right] \end{aligned} \quad (2.10a)$$

$$= MV_{oR}^2 \left[1 - \frac{1}{2n} \right] \quad (2.10b)$$

$$\text{where } n = \frac{r}{R}$$

c) Launch from orbit r to escape

$$E_{K_1} = \frac{\gamma M M_p}{2r} \quad E_{K_2} = 0$$

$$E_{p_1} = -\frac{\gamma M M_p}{r} \quad E_{p_2} = 0$$

$$= -MV_{or}^2$$

$$\begin{aligned} \text{So } \Delta E_m &= [E_{K_2} + E_{p_2}] - [E_{K_1} + E_{p_1}] \\ &= \frac{\gamma M M_p}{2r} \end{aligned} \quad (2.11a)$$

$$= \frac{1}{2} M V_{or}^2 \quad (2.11b)$$

d) The energy requirement for launch from surface to escape:

From equation (2.10) above, when $r \rightarrow \infty$, $n \rightarrow \infty$

$$\Delta E_m = \frac{\gamma M M_p}{R} = M V_{oR}^2$$

The same result is obtained when equation 2.10 and 2.11 are added

since $V_{or}^2 = V_{oR}^2 / n$

- e) The energy requirement for changing from orbit at radius r_1 to orbit at radius r_2 is:

$$\begin{aligned}\Delta E_m &= \left[\frac{\gamma M M_p}{2 r_2} - \frac{\gamma M M_p}{r_2} \right] - \left[\frac{\gamma M M_p}{2 r_1} - \frac{\gamma M M_p}{r_1} \right] \\ &= \frac{\gamma M M_p}{2 r_1} \left[1 - \frac{1}{n} \right], \quad n = \frac{r_2}{r_1}\end{aligned}\tag{2.12}$$

A useful rearrangement of this equation to make use of the potential well concept is:

$$\begin{aligned}\frac{\Delta E_m}{M} &= \frac{\gamma M_p}{2 r_1} \left[1 - \frac{1}{n} \right] = \frac{\gamma M_p}{R} \cdot \frac{R}{2 r_1} \left[1 - \frac{1}{n} \right] \\ &= P \cdot \frac{R}{2 r_1} \left[1 - \frac{1}{n} \right]\end{aligned}\tag{2.12a}$$

where $P = \frac{\gamma M_p}{R}$ = the potential well given in Table 1 for the particular body of interest.

R = the radius of the body generating the potential well.

Data for one-way, orbit-to-orbit, interplanetary missions originating at the earth derived from these equations and the data from Table 1 are tabulated in Table 2. An example, of the calculation for transfer from EARTH orbit to PLUTO orbit is given below:

- a) Note that the potential well of the SUN is much greater than for any of the planets, so, to a first approximation, the problem can be treated as a two-body problem involving the vehicle and the SUN. Therefore, we use the potential well of the SUN and consider the orbit-to-orbit change as being from the EARTH-orbit radius to the PLUTO-orbit radius.
- b) From Table 1, $P = 1.9 \times 10^{11}$ joules/(kg) for the SUN, and the SUN's radius, $R = 6.96 \times 10^8$ m.
- c) From Table 2, the EARTH orbit radius, $r_1 = 1.497 \times 10^{11}$ m, and the PLUTO orbit radius, $r_2 = 59.15 \times 10^{11}$ m. Therefore

$$n = \frac{r_2}{r_1} = \frac{59.15 \times 10^{11}}{1.497 \times 10^{11}} = 39.5$$

TABLE 1

Potential Wells in the Solar System

Object	Mass M_p (Kg)	Radius R (m)	Well Depth		Escape Velocity, V_e (m/sec) (ft/sec)
			$\frac{\Delta E_m}{g_0 m}$ (joules/Kg)		
Sun	1.99×10^{30}	6.96×10^8	1.91×10^{11}	6.18×10^5	2,020,000
Mercury	3.17×10^{23}	2.42×10^6	8.74×10^6	4.18×10^3	13,730
Venus	4.87×10^{24}	6.20×10^6	5.24×10^7	1.02×10^4	33,400
Earth	5.98×10^{24}	6.38×10^6	6.25×10^7	1.12×10^4	36,700
Mars	6.40×10^{23}	3.40×10^6	1.25×10^7	5.0×10^3	16,400
Jupiter	1.90×10^{27}	7.14×10^7	1.77×10^9	5.97×10^4	196,000
Saturn	5.69×10^{26}	6.04×10^7	6.28×10^8	3.54×10^4	116,000
Uranus	8.70×10^{25}	2.38×10^7	2.44×10^8	2.2×10^4	72,100
Neptune	1.03×10^{26}	2.23×10^7	3.08×10^8	2.48×10^4	81,300
Pluto	5.4×10^{24}	7.2×10^6	5.0×10^7	1.0×10^4	32,800
Moon	7.34×10^{22}	1.74×10^6	2.81×10^6	2.37×10^3	7,760
Phobos	1×10^{15}	8×10^3	8.3×10^1	4×10^0	13
Ceres	8×10^{20}	3.7×10^5	1.4×10^5	5×10^2	1,640
Eros	1×10^{16}	1×10^4	6.67×10^1	1×10^1	33
Pallas	2×10^{20}	2.4×10^5	5.5×10^4	3×10^2	983

Note:

$$V_e = \sqrt{\frac{2 g_0 \frac{\Delta E_m}{g_0 M}}{2.985}} \quad (\text{ft/sec})$$

where

$$g_0 = \text{ft/sec}$$

$$\frac{\Delta E_m}{g_0 M} = \text{joules/Kg}$$

TABLE 2Energy Requirement for One-way, Orbit-to-Orbit, Interplanetary Missions
Originating at the EARTH^a

Target Planet	Radius from Sun $r \times 10^{11} \text{ (m)}$	Ratio of Planet Orbit Radius to Earth Orbit Radius n	$E_m \times 10^{-8}$ $m g_0$ (joule/Kg)	Time for Transfer (yr)
Mercury	0.579	0.387	7.01	0.289
Venus	1.083	0.723	1.70	0.400
Earth	1.497	1.000	-	-
Mars	2.281	1.524	1.53	0.709
Jupiter	7.787	5.20	3.59	2.732
Saturn	14.28	9.54	3.98	6.05
Uranus	28.72	19.19	4.21	16.12
Neptune	45.01	30.07	4.29	30.6
Pluto	59.15	39.5	4.33	45.5

^a For Hohmann-type transfer orbits between coplanar, circular planetary orbits.

d) Hence, from equation 2.12a

$$\begin{aligned}\frac{\Delta E_m}{M} &= P \cdot \frac{R}{2r_1} \left[1 - \frac{1}{n} \right] \\ &= 1.91 \times 10^{11} \times \frac{6.96 \times 10^8}{2 \times 1.497 \times 10^{11}} \left[1 - \frac{1}{39.5} \right] \\ &= 4.33 \times 10^8 \text{ joules/kg}\end{aligned}$$

This corresponds to the value given for PLUTO in Table 2. The total energy that must be imparted to the vehicle for launching from the EARTH's surface and landing on the surface of PLUTO is, to the first approximation

$$\begin{aligned}\left[\frac{\Delta E}{g_0 M} \right]_m &= \left| P_E \right| + \left| \frac{\Delta E}{g_0 M} \right| + \left| P_p \right| \\ &= 6.25 \times 10^7 + 4.33 \times 10^8 + 5.0 \times 10^7 \\ &= 5.45 \times 10^8 \text{ joules/kg} = 249 \frac{\text{MW sec}}{\#}\end{aligned}$$

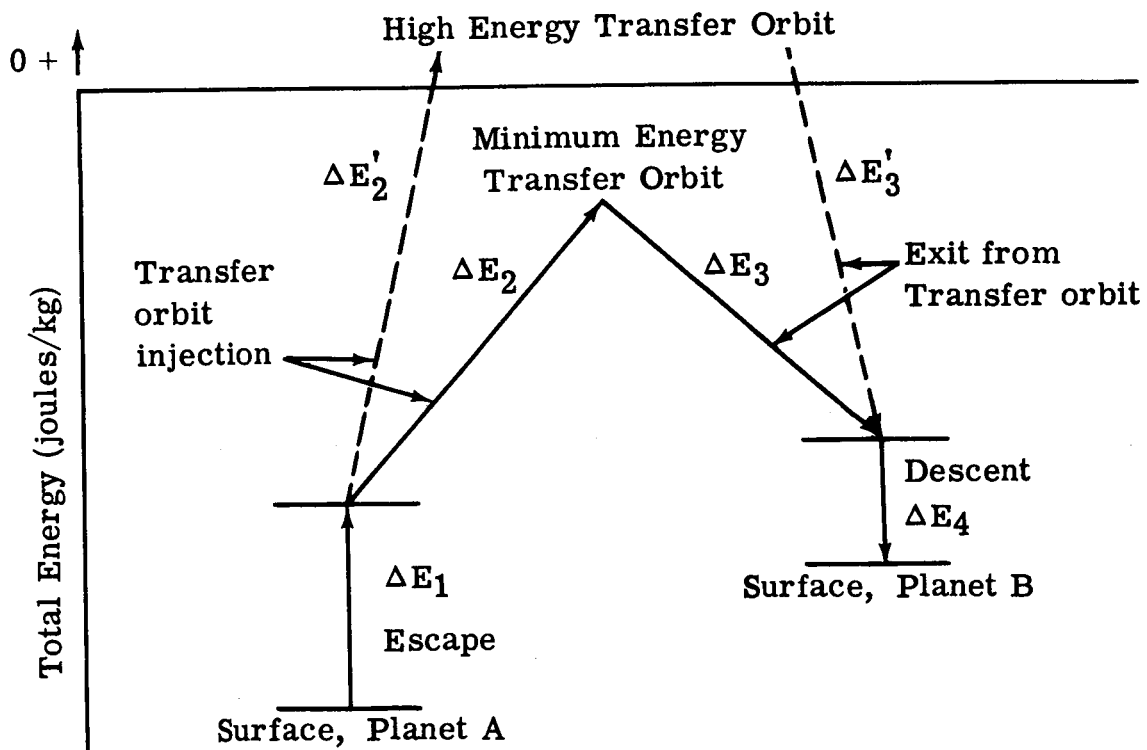
where P_E = Potential well for EARTH

P_p = Potential well for PLUTO

$\frac{\Delta E}{g_0 M}$ = Transfer from EARTH orbit to PLUTO orbit.

This assumes that upon landing on PLUTO, energy is expended to brake the vehicle to avoid acceleration due to PLUTO's gravitational field. On this basis, the same energy is expended to make the return mission from PLUTO to EARTH.

The information presented to this point represents the energy increment that must be imparted to the vehicle just sufficient to complete the mission, denoted by ΔE_m . For shorter duration missions, higher energy increments can be imparted to the vehicle, denoted by ΔE_v . The differences are indicated by the following sketch.



Minimum Energy: $\Delta E_m = \Delta E_1 + \Delta E_2 + \Delta E_3 + \Delta E_4$

High Energy: $\Delta E_v = \Delta E_1 + \Delta E'_2 + \Delta E'_3 + \Delta E_4$

The actual energy supplied by the reactor heat source, ΔE_{ps} , is generally different from either ΔE_m or ΔE_v because of the kinetic energy of the propellant before and after the pulse.*

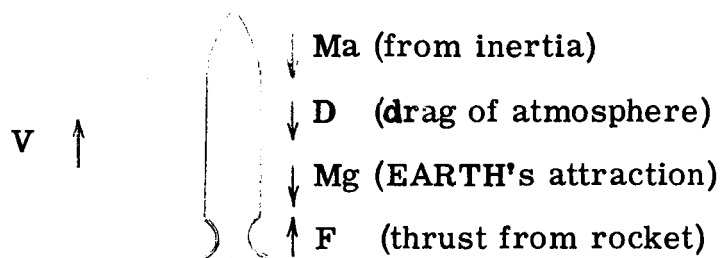
*In some cases, the reactor energy increment can be less than the vehicle energy increment imparted during a single pulse. This is possible because the ejected propellant contained kinetic energy before the pulse; see Appendix A-1 for further discussion.

2.2 Reactor Energy Requirement

The most direct approach for determining the necessary reactor increment involves making a force balance on the vehicle, then finding the velocity increment achievable as a function of propellant consumption. The velocity increment is a measure of the useful energy imparted to the vehicle, and the propellant consumption is a measure of the energy required. In all work that follows, a single stage rocket is considered.

2.2.1. Basic Rocket Equation for a Single Stage

The force balance is:*



The equation of motion for the vehicle is:

$$F - Mg - D = Ma = M \frac{dv}{dt} \quad (2.13)$$

For constant propellant velocity with respect to the vehicle and constant flow rate, the thrust given by the propellant is:

$$F = \dot{M}_P V_j \quad (2.14)$$

where

$\dot{M}_P = dm/dt$ = flow rate of propellant

V_j = jet velocity with respect to the vehicle.

Equations (2.13) and (2.14) may be combined and integrated:

$$\dot{M}_P V_j - Mg - D = M \frac{dv}{dt}$$

or

$$\int dv = \dot{M}_P V_j \int_0^t \frac{dt}{M} - \int_0^t g dt - \int_0^t \frac{D dt}{M} \quad (2.15)$$

* Zucrow, M. J., Aircraft and Missile Propulsion, Vol. II, p. 564, J. Wiley & Sons, Inc., 1958.

The mass of the vehicle at any time t is $M = M_0 - \dot{M}_p t$,

so

$$\dot{M}_p V_j \int_0^t \frac{dt}{M} = V_j \int_0^t \frac{\dot{M}_p dt}{M_0 - \dot{M}_p t} = -V_j \ln \left[\frac{(M_0 - \dot{M}_p t)}{M_0} \right]$$

$$= V_j \ln \frac{M_0}{M}$$

Therefore

$$\Delta V = V_2 - V_1 = V_j \ln \frac{M_0}{M} - \int_0^t g dt - \int_0^t \frac{D dt}{M_0 - \dot{M}_p t}$$

If the vehicle is programmed with time so as to tilt at the angle θ from the horizon, the term g should be multiplied by $\sin \theta$; also, g is a function of the distance from the EARTH's surface, so:

$$\int_0^t g dt = \int_0^t g_0 \left[\frac{R}{R+r} \right]^2 \sin \theta dt = \bar{g} t$$

where

R = EARTH's radius at which $g = g_0$

r = height above EARTH's surface.

The effect of drag may be treated by calculating an average drag \bar{D} over the flight profile through the atmosphere so that:

$$\int_0^t \frac{D dt}{M_0 - \dot{M}_p t} = - \frac{\bar{D}}{\dot{M}_p} \int_0^t \frac{\dot{M}_p dt}{M_0 - \dot{M}_p t} = V_j \frac{\bar{D}}{\dot{M}_p} \ln \frac{M_0}{M}$$

and, then:

$$\Delta V = V_j \left[1 - \frac{\bar{D}}{\dot{M}_p} \right] \ln \frac{M_0}{M} - \bar{g} t \quad (2.16)$$

where: t is the time of thrust application.

For large rockets, 50 tons or more at take-off, the effect of drag is relatively small and reduces the cut-off velocity only a few percent.* Of course, for orbital changes not involving launch through the atmosphere, the drag term is zero. The effect of the time of thrust application, on the other hand, is never negligible; the magnitude of the effect may be appreciated by considering the two possible extremes of energy required for escape from a planet's gravity field as determined by whether the thrust is a short instantaneous pulse or is continuous throughout an increasing spiral flight profile. For instantaneous impulse, from equation 2.9 escape velocity, $V_e = \sqrt{2} V_0$, so:

$$\Delta V = V_0 \left[\sqrt{2} - 1 \right].$$

For continuous thrust, the equivalent velocity increments is

$$\Delta V = V_0$$

as shown by the derivation in Appendix A-2.⁺ The ratio of the velocity increment is then:

$$\frac{\Delta V(\text{continuous})}{\Delta V(\text{impulse})} = \frac{V_0}{V_0 \left[\sqrt{2} - 1 \right]} = 2.4$$

Significantly higher energy inputs are necessary, therefore, for long-time thrust applications.

2.2.2 Energy Supplied to the Propellant

Finally, an indication of the energy input to the propellant may be obtained by considering the case of short-time thrust application such that

$$\Delta V \gg \bar{g}t$$

* Ibid., p. 568-570.

+ Per Corliss, p. 47 - In the actual case, for different times of thrust application, equation 2.15 should be integrated for the particular flight profile desired.

so that

$$\Delta V = V_j \ln \frac{M_o}{M}$$

Since the mass change is due entirely to the expulsion of propellant, then the quantity of propellant and the energy input can be found. That is:

$$Q_{in} = [M_o - M] g_o \Delta h = \text{heat to propellant from reactor} \quad (2.18)$$

where Δh is the enthalpy rise of the propellant due to the heat added by the reactor.

From the general equation for expansion in a nozzle, it can be shown that for complete expansion to a vacuum*:

$$V_j^2 = 2 g_o \Delta h \quad (2.19)$$

So

$$\frac{Q_{in}}{M} = \left[\frac{M_o}{M} - 1 \right] \frac{V_j^2}{2} = \left[e^{\Delta V/V_j} - 1 \right] \frac{V_j^2}{2} \quad (2.18a)$$

or in dimensionless terms:

$$\frac{Q_{in}}{M \Delta V^2} = \left[e^{\frac{\Delta V}{V_j}} - 1 \right] \left(\frac{V_j/\Delta V}{2} \right)^2 \quad (2.20)$$

The term Q_{in}/M represents the propellant energy input per unit of mass at burnout, and is a measure of the overall efficiency required to achieve a particular ΔV , under the ideal condition that:

*This assumes infinite nozzle discharge area. See Bussard and DeLauer, pp. 18-42 for theoretical performance data for real chemical systems.

- a) Atmospheric drag is zero.
- b) The time of thrust application is negligible.
- c) All the heat energy of the propellant is converted to kinetic energy in the jet.

There is an optimum propellant mass fraction which minimizes the heat input required for a given ΔV as shown in Table 3, below:

TABLE 3

Mass Ratio M_o/M_f	Propellant Fraction $(M_p/M_o) \times 100\%$	$\frac{\Delta V}{V_j}$	$\frac{Q_{in}}{M_p(\Delta V)^2}$
1000	99.9%	6.91	10.46
100	99	4.61	2.33
10	90	2.30	0.834
4.93*	80	1.595	0.774
2	50	0.694	1.040
1	0	0	---

*Optimum value.

This indicates that: (a) the propellant fraction for any particular energy impulse should be about 80% of the total vehicle initial mass; and (b) the ~~minimum~~ reactor energy necessary is less than double the energy increment when the propellant fraction is between 50 and 90% the initial vehicle mass.

2.2.3 Minimum Velocity Increments for Interplanetary Missions

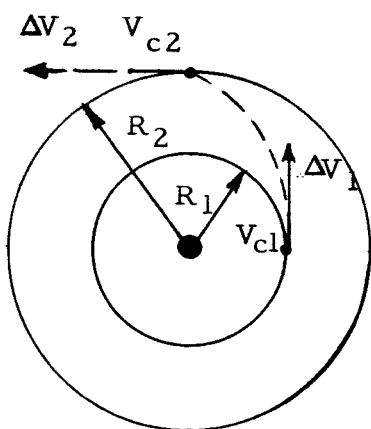
At this point, the propellant energy requirement can be established once the mission total velocity increment is fixed. Precise determination of trajectories and impulse velocity increments for particular missions is an entire subject within itself, but a reasonable "feel" for minimum requirements for interplanetary missions can be obtained by use of the basic information presented to this point combined with several simplifying assumptions, i. e.:

1. Velocity increments are achieved by short pulses at the proper time and direction to eliminate the need for any significant in-flight corrections.
2. The various planets of the solar system are in circular, coplanar orbits about the sun at the radii listed in Table 2.
3. All orbit transfers are accomplished by the minimum energy (Hohmann) trajectory.
4. Where possible, full use will be made of the planet's gravity field to minimize the required velocity increment.

Data generated on this basis are given in Figures 1 through 4. Except for the launch from earth to low orbit (Figure 1), or landings on the target planet (Figure 4), some further explanation is required.

a) Transfer from earth orbit to target planet.

The initial impulse is given while in low orbit to take advantage of the initial velocity; sufficient energy is added during the pulse to escape earth's gravity with an excess velocity equal to that required for the Hohmann type elliptical transfer (in accordance with Cooper's analysis, LAMS 2632). The Hohmann transfer between two circular orbits is done in two pulses as indicated below:



$$\Delta V_1 = \left(\sqrt{\frac{2n}{1+n}} - 1 \right) V_{c1}$$

$$\Delta V_2 = \left(1 - \sqrt{\frac{2}{1+n}} \right) V_{c2}$$

$$= \left(1 - \sqrt{\frac{2}{1+n}} \right) \frac{V_{c1}}{\sqrt{n}}$$

where: V_{c1} = circular orbit velocity at radius, R_1 , with respect to the sun.

V_{c2} = circular orbit velocity at radius, R_2 , with respect to the sun.

$$n = \frac{R_2}{R_1}$$

MISSION: EARTH LAUNCH TO ORBIT AT $r = r_0$
(NO DRAG OR TIME OF PULSE EFFECTS)

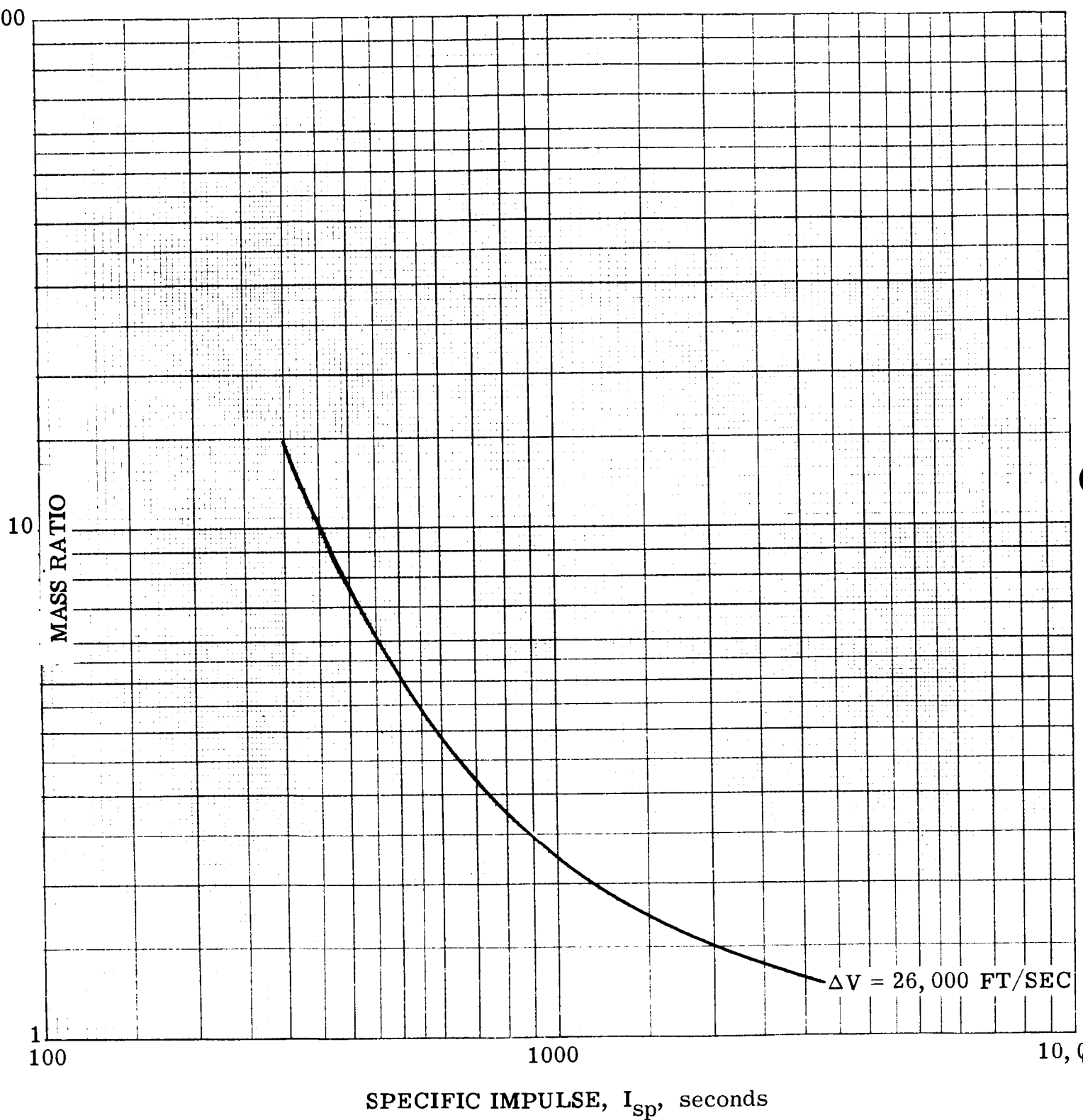


FIGURE 1

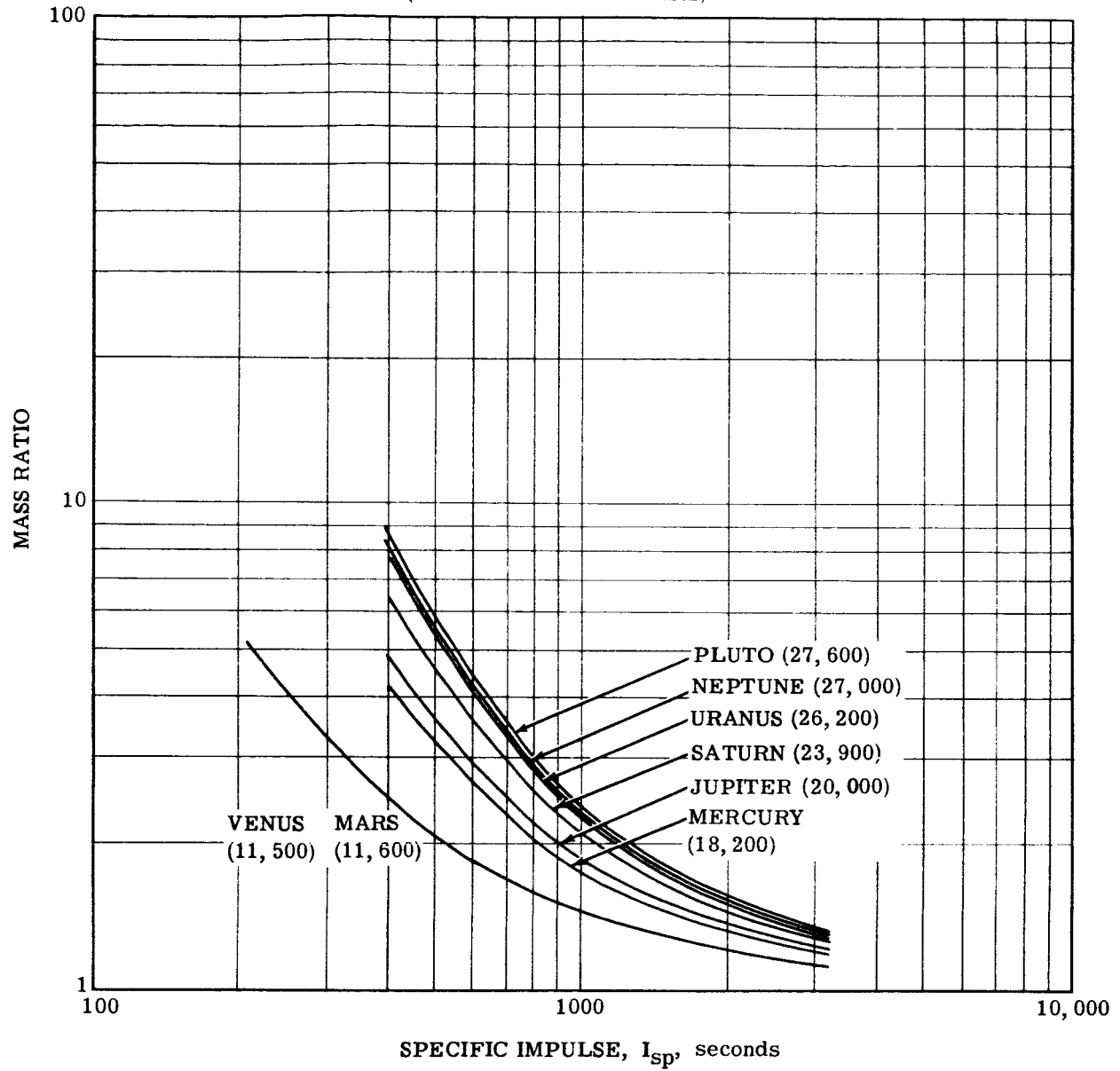


FIGURE 2

MISSION: ORBIT PLANET FROM FLYBY POSITION (ΔV_2 OF HOHMANN
ELLIPSE MODIFIED BY USING PLANETARY GRAVITY FIELD)

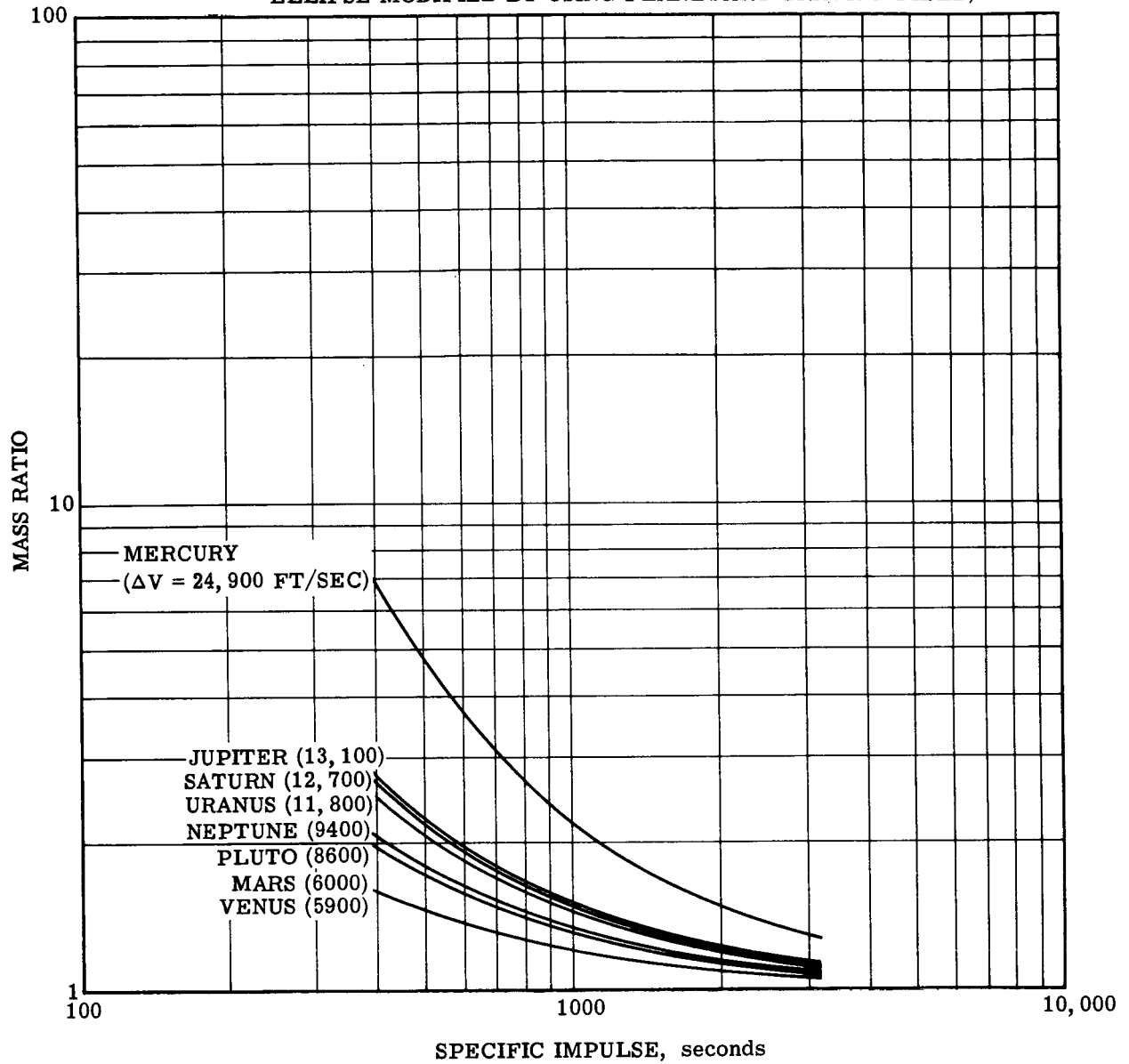


FIGURE 3

MISSION: PLANETARY LANDING FROM ORBIT ESTABLISHED
BY MINIMUM ENERGY CAPTURE

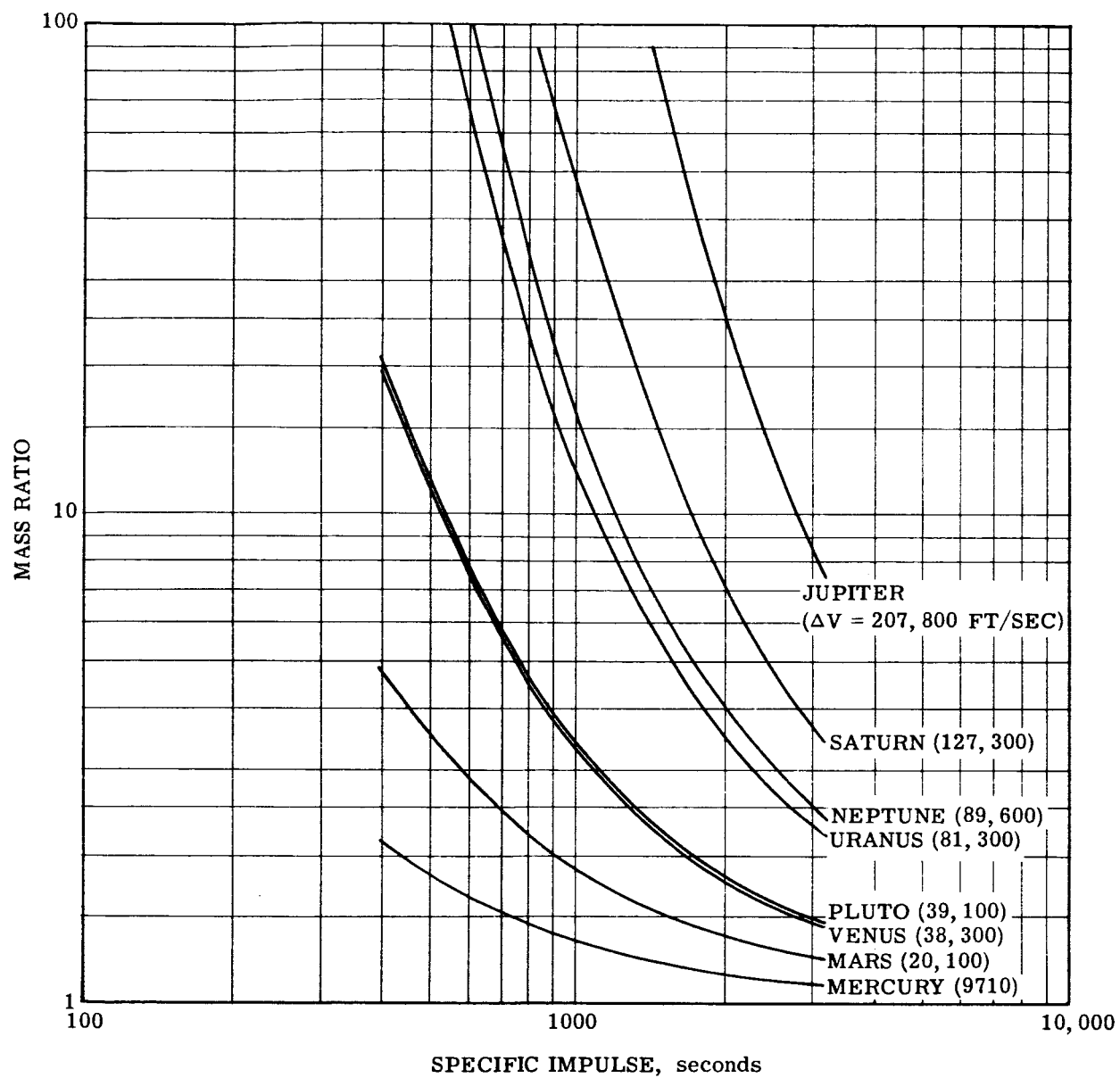


FIGURE 4

When the vehicle is in circular orbit at velocity, v_o , about a planet at radius r_1 , additional energy must be supplied to escape the planet's potential well. Cooper has shown that a smaller total ΔV is required if the escape and orbit transfer pulses are combined rather than done separately. That is, to escape from the planet only with a first pulse:

$$\Delta V_{\text{esc}} = (\sqrt{2} - 1) v_{o1}$$

Adding the second pulse, ΔV_1 , for the first step of the Hohmann transfer we get the total

$$\begin{aligned} \Delta V_T &= \Delta V_{\text{esc}} + \Delta V_1 \\ &= (\sqrt{2} - 1) v_{o1} + \left(\sqrt{\frac{2n}{1+n}} - 1 \right) V_{c1} \end{aligned}$$

On the other hand, if the first pulse is greater than required merely for escape, the energy balance shows:

$$\frac{V_h^2}{2} - \frac{\gamma M_p}{r_o} = \frac{V_{\text{ex}}^2}{2}$$

where $V_h = v_{o1} + \Delta V_1^1$ = velocity immediately after the pulse.

V_{ex} = velocity after escaping the planet's gravity field, with respect to the planet.

$$\begin{aligned} \text{or } V_h &= \sqrt{V_{\text{ex}}^2 + \frac{2 \gamma M_p}{r_o}} \\ &= \sqrt{V_{\text{ex}}^2 + 2 v_{o1}^2} \end{aligned}$$

$$\text{and } \Delta V_1^1 = V_h - v_{o1} = \sqrt{V_{\text{ex}}^2 + 2 v_{o1}^2} - v_{o1}$$

In order to make the first step of the Hohmann transfer, a necessary condition is that

$$V_{ex} = \left(\sqrt{\frac{2n}{1+n}} - 1 \right) V_{c1}$$

$$\text{Hence } \Delta V_T^1 = \Delta V_1^1 = \sqrt{\left(\sqrt{\frac{2n}{1+n}} - 1 \right)^2 V_{c1}^2 + 2 v_{o1}^2} - v_{o1}$$

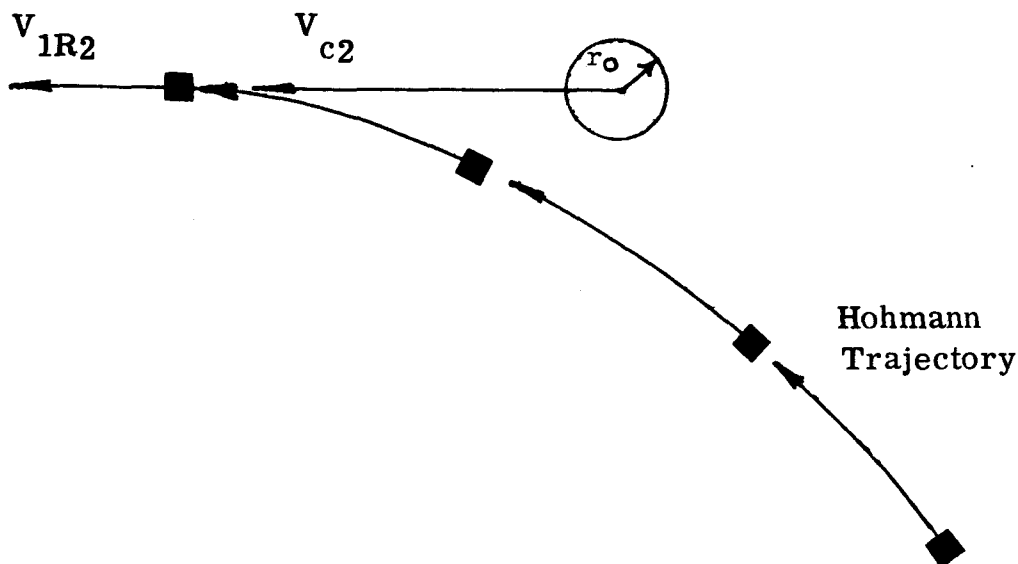
and it is easily shown that $\Delta V_T^1 < \Delta V_T$. Therefore, the first pulse should be sufficiently large so that the residual velocity, V_{ex} , with respect to the planet (after escaping the planet's gravity field) is equal to the required velocity with respect to the sun for the first step of the Hohmann ellipse.

b) Establishing a circular orbit coincident with the target planet's orbit.

If the target planet is far removed from the point at which the elliptical trajectory of the vehicle becomes tangent to the orbit of the second planet about the sun, R_2 , then the full velocity increment given by ΔV_2 must be applied. On the other hand, by aiming at the proper orbit radius, r_2 , about the target planet, the need for a second velocity increment can be reduced.

Cooper has shown the analysis for the case in which:

$$\Delta V_2^1 = \sqrt{\Delta V_2^2 + 2 v_o^2} - v_o,$$



For the Hohmann transfer

$$V_{c2} - V_{1R2} = \Delta V_2$$

But the excess velocity of the vehicle approaching the planet, V_{ex} , is also $V_{c2} - V_{1R2}$; so, in the energy equation we get

$$\frac{V_h^2}{2} - \frac{\gamma M_p}{r_2} = \frac{V_{ex}^2}{2} = \frac{(\Delta V_2)^2}{2}$$

$$\text{or } V_h = \sqrt{(\Delta V_2)^2 + \frac{2 \gamma M_p}{r_2}}$$

and

$$\begin{aligned} \Delta V_2^1 &= V_h - v_{o2} = V_h - \sqrt{\frac{\gamma M_p}{r_2}} \\ &= \sqrt{(\Delta V_2)^2 + 2 \frac{\gamma M_p}{r_2}} - \sqrt{\frac{\gamma M_p}{r_2}} \end{aligned}$$

Cooper shows ΔV_2^1 is a minimum when

$$r_2 = \frac{2 \gamma M_p}{(\Delta V_2)^2}$$

$$\text{or } \frac{r_2}{r_o} = 2 \left(\frac{v_o}{\Delta V_2} \right)^2 = \left(\frac{v_{esc}}{\Delta V_2} \right)^2$$

$$\begin{aligned} \text{If } \frac{r_2}{r_o} \geq 1, \quad \Delta V_2^1 &= \sqrt{2} \Delta V_2 - \frac{\Delta V_2}{\sqrt{2}} \\ &= 0.707 \Delta V_2 \end{aligned}$$

Otherwise, ΔV_2^1 is a minimum when $r_2 = r_0$; the maneuver may be interpreted as the opposite of the Earth launch case.

By way of illustration of the importance of the use of these concepts for minimizing the velocity increment requirements for interplanetary maneuvers, Table 4A lists pertinent data for the simple Hohmann transfer, in which the velocity impulse of 10,700 ft/sec for earth escape has already been supplied; while, Table 4B lists corresponding data when the required pulses are combined. The data in Figures 2 and 3 assume the minimum values for ΔV as tabulated in Table 4B.

2.2.4 Propellant Energy Requirements for Interplanetary Missions

Using the data in Figures 1 through 4, the propellant energy increment is fixed as given in Tables 5A and 5B: (1) for the case in which the initial-to-final mass ratio of 5 is arbitrarily assumed* so that the specific impulse (Isp)⁺ necessary is fixed; and, (2) for the case in which a specific impulse of 800 seconds is arbitrarily assumed so that the initial-to-final mass ratio is fixed. Specific missions of interest included in the table are described briefly below:

- a) Orbit transfer from Earth to the target planet - These data apply for the case in which the nuclear rocket is already in orbit about the Earth, having been launched by any suitable means. The mission essentially is a planetary flyby following a Hohmann ellipse, with the target planet in optimum position with respect to the Earth. This mission imposes the smallest requirement on the rocket engine; for the nuclear rocket with a specific impulse of 800 seconds, initial-to-final mass ratios appreciably less than 5 are possible.
- b) Orbit of target planet - This simply adds the second impulse of the Hohmann transfer, making use of the planet's gravity field to minimize the requirement. A specific impulse of 800 seconds permits orbiting any planet with a mass ratio of 5 or less (Mercury requires a fraction more than 5), provided the vehicle starts from orbit about Earth; for Mars and Venus, the necessary mass ratio is less than two, showing that non-optimum missions may reasonably be performed.

* It should be noted that when the mass ratio of 5 is assumed, the least reactor energy input for a given velocity increment is required and sufficient margin for payload is provided even if the tank weight fraction should be as high as 10% of the propellant weight.

⁺ Specific impulse is defined by $I_{sp} = \text{lb thrust/lb flow/sec} = \text{seconds}$.

TABLE 4A

Mission Requirements for One-Way, Orbit-to-Orbit Transfer to Other Planets
from Earth after Escaping Earth's Gravity Field^a

Target Planet	Radius from Sun	Ratio of Orbit Radii: Planet/Earth	Energy Increment		Velocity	Increments ^b		Time for a Transfer
			$\Delta E \times 10^{-8}$					
			$\frac{g_o}{M}$			$\frac{\Delta V_1^c}{\text{(ft/sec)}}$	$\frac{\Delta V_2}{\text{(ft/sec)}}$	
			$r \times 10^{-11}$ (meters)	n			t (year)	
-	-	-	-	-	-	-	-	-
Mercury	0.579	0.387	7.01		24730	31,700	0.289	
Venus	1.083	0.723	1.70		8210	8,300	0.400	
Earth	1.497	1.000	-		-	-	-	
Mars	2.281	1.524	1.53		9580	8,500	0.709	
Jupiter	7.787	5.20	3.59		28850	18,500	2.732	
Saturn	14.28	9.54	3.98		33740	17,900	6.05	
Uranus	28.72	19.19	4.21		37180	15,300	16.12	
Neptune	45.01	30.07	4.29		38250	13,300	30.6	
Pluto	59.15	39.5	4.33		38820	12,100	45.5	

^a Note that the velocity increment of 10,700 to escape Earth is not included in ΔV_1 .

^b For Hohmann transfer.

^c Assuming that vehicle is orbiting about the Sun at Earth-orbit radius, but is free of the Earth's gravity field.

TABLE 4B

Mission Requirements for One-Way, Orbit-to-Orbit Transfer to Other Planets from Earth, Starting from Earth Orbit and Ending in Planetary Orbit

Target Planet	Energy Increment	Velocity		Increments ^a	Optimum Orbit Radius to achieve ^b Minimum ΔV_2	
		$\Delta E \times 10^{-8}$ $\frac{g_0 M}{m}$	ΔV_1 (ft/sec)		ΔV_2 (ft/sec)	r/r_0
-						
-						
Mercury	7.41		18,200	24,900		1
Venus	2.33		11,500	5,900		16.2
Mars	1.87		11,600	6,000		3.73
Jupiter	4.06		20,000	13,100		112.
Saturn	4.31		23,900	12,700		42.
Uranus	4.53		26,200	11,800		22.2
Neptune	4.68		27,000	9,400		37.4
Pluto	4.71		27,600	8,600		7.35

a For Hohmann transfer, per method in LAMS-2632 (Cooper).

b Assumes use of target planet's gravity field.

Note. All other factors in Table 4A not listed above are the same.

c) Landing on the target planet - The velocity increment required for a soft-landing has been added to that required in item (b). To some extent, performing this mission without staging is academic in view of the fact that significantly different thrust-to-weight ratio rockets will be required to make soft landings, and it is likely that if use of nuclear engines is avoided for the boost phase from Earth, it would also be avoided when landing on other planets. Nevertheless, the data indicate which planets are reasonably within reach; Mars is the prime candidate for a specific impulse of 800 seconds, but Mercury and Venus are also within reach.

d) The missions a, b, and c, above, may also be started from Earth's surface, and data for these cases are also tabulated. No planetary landing missions are possible in this case without exceeding a mass ratio of 10; while the mars landing is marginal, needing a mass ratio of 11.8 for 800 second specific impulse, other planets require in excess of 20, which does not allow sufficient margin for the propellant tank weight fraction. Venus, Mars, or Jupiter can be orbited, however, with a single stage starting from Earth's surface. When and if nuclear rockets can develop 3000 seconds specific impulse, surface to surface missions appear possible for all planets except Jupiter and Saturn.

It must be emphasized that the velocity increments on which these results are based are the absolute minimum required for the mission; higher velocity increments will be necessary to account for atmospheric drag, time of pulse application, for decreasing transit time, and for performing such mission at non-optimum times with respect to relative planet locations. A number of sources in the literature are available including off-optimum, non-ideal conditions*.

* "Propulsion for Interplanetary Space Missions", F. M. Kirby, IAS Paper No. 62-85, Jan. 1962; or, "Nuclear Rocket Applications", Paul G. Johnson, Astronautics, Dec., 1962, pp. 22-27.

TABLE 5A

Requirements for Interplanetary Missions:
Specific Impulse and Energy Required for Mass Ratio of 5

Planet	Starting from Earth Orbit			Starting from Surface of Earth ^d		
	Flyby ^a $\frac{I_{sp}}{Q_{in}} \frac{M_f}{M_i}$ (sec) MW hr Kg	Orbit ^b $\frac{I_{sp}}{Q_{in}} \frac{M_f}{M_i}$ (sec) MW hr Kg	Land ^c $\frac{I_{sp}}{Q_{in}} \frac{M_f}{M_i}$ (sec) MW hr Kg	Flyby $\frac{I_{sp}}{Q_{in}} \frac{M_f}{M_i}$ (sec) MW hr Kg	Orbit $\frac{I_{sp}}{Q_{in}} \frac{M_f}{M_i}$ (sec) MW hr Kg	Land $\frac{I_{sp}}{Q_{in}} \frac{M_f}{M_i}$ (sec) MW hr Kg
Mercury	350 .0066	830 .0369	1020 .0557	855 .0390	1320 .0922	1520 .1239
Venus	220 .0026	336 .0060	1075 .0616	725 .0280	837 .0375	1575 .1330
Mars	224 .0027	340 .0062	710 .0268	730 .0285	840 .0380	1230 .0807
Jupiter	390 .0081	640 .0219	4650 1.158	880 .0414	1140 .0697	5150 1.420
Saturn	470 .0118	710 .0268	3160 .534	960 .0500	1210 .0783	3660 .718
Uranus	510 .0139	735 .0393	2300 .284	1000 .0534	1235 .0815	2800 .420
Neptune	530 .0150	700 .0376	2430 .316	1000 .0534	1205 .0777	2930 .460
Pluto	540 .0156	700 .3074	1450 .113	1000 .0534	1200 .0772	1950 .204

^a Assumes first step of Hohmann minimum energy elliptical transfer is applied from low Earth orbit.

^b Assumes a plus second step of Hohmann type impulse, using the planet's gravity field to enter optimum orbit radius.

^c Assumes a plus landing operation.

^d Same as a, b, and c but adding launch from Earth to each.

TABLE 5B

Requirements for Interplanetary Missions:
Mass Ratio and Energy Required for Specific Impulse of 800 sec.

Planet	Starting from Earth Orbit				Starting from Surface of Earth			
	Flyby $\frac{M_o}{M_f} \frac{Q_{in}}{M_f}$ - MW hr Kg	Orbit $\frac{M_o}{M_f} \frac{Q_{in}}{M_f}$ - MW hr Kg	Land $\frac{M_o}{M_f} \frac{Q_{in}}{M_f}$ - MW hr Kg	Flyby $\frac{M_o}{M_f} \frac{Q_{in}}{M_f}$ - MW hr Kg	Orbit $\frac{M_o}{M_f} \frac{Q_{in}}{M_f}$ - MW hr Kg	Land $\frac{M_o}{M_f} \frac{Q_{in}}{M_f}$ - MW hr Kg		
Mercury	2.03 .0089	5.35 .0374	7.78 .0582	5.57 .0392	14.7 .1175	21.3 .1745		
Venus	1.56 .0048	1.97 .0083	8.68 .0660	4.29 .0282	5.39 .0376	23.8 .1958		
Mars	1.57 .0049	1.98 .0084	4.31 .0284	4.30 .0283	5.45 .0382	11.8 .0928		
Jupiter	2.17 .0100	3.62 .0225	11500 .98.8	5.97 .0426	10.0 .0773	62000 .533		
Saturn	2.53 .0131	4.15 .0270	580 .4.97	6.95 .0510	17.4 .0893	1680 .13.74		
Uranus	2.76 .0150	4.37 .0289	198 .876	7.58 .0565	12.0 .0945	281 2.407		
Neptune	2.85 .0158	4.10 .0266	184 1.142	7.82 .0585	11.3 .0885	385 3.128		
Pluto	2.92 .0164	4.09 .0265	18.6 .1511	8.01 .0603	17.2 .0875	51.0 .4293		

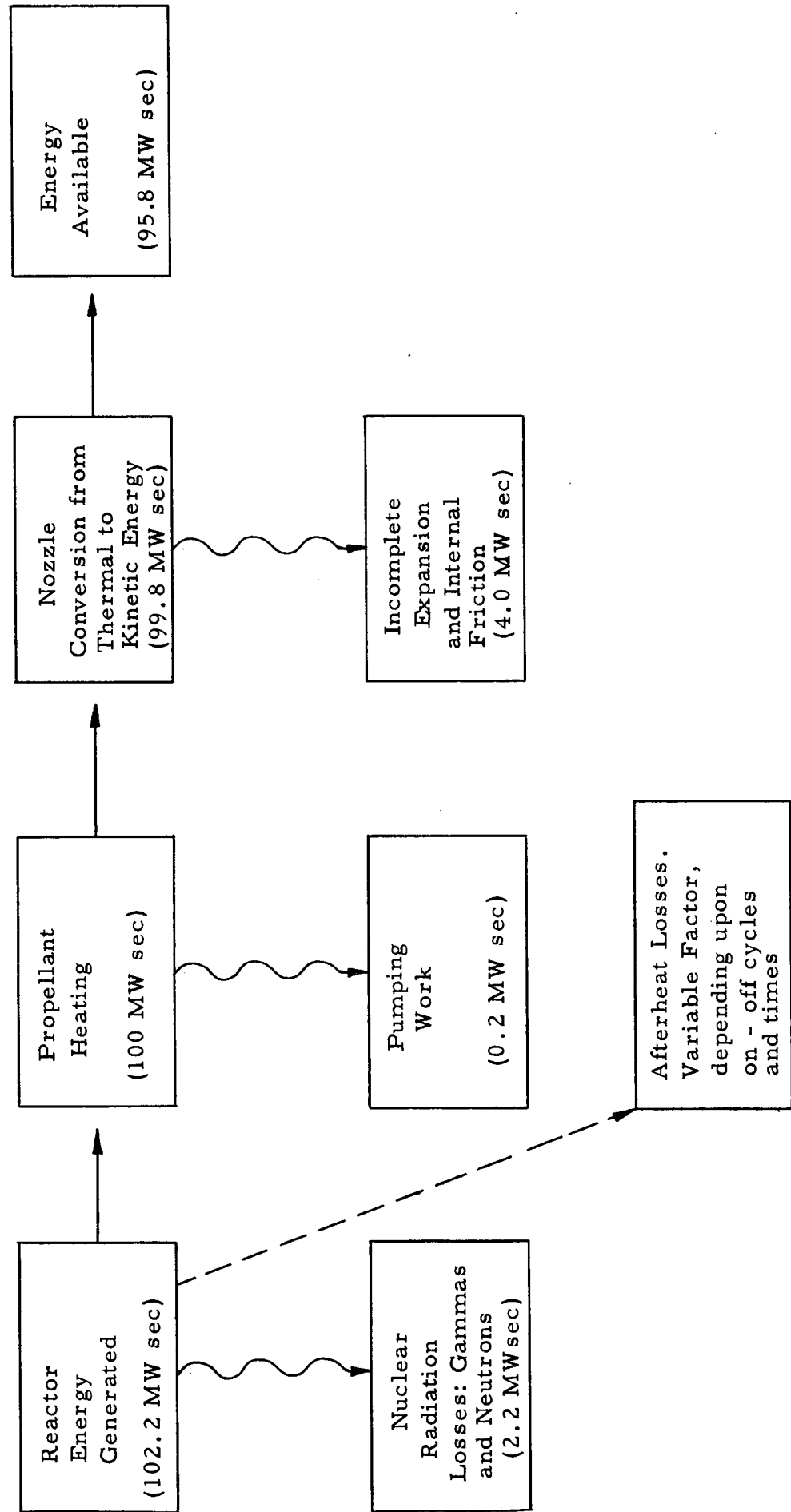
2.2.5 Thermal Efficiency

The thermal efficiency of a nuclear rocket engine may be considered as consisting of the terms shown in Figure 5 . For calibration, a rough estimate of losses to each parasite source is given: the only significant loss occurs in the nozzle and in the reactor; so far as the expenditure of propellant is concerned, the nozzle loss and the afterheat losses are the only ones of concern. The magnitude of the losses are small in either event, and the thermal efficiency is very high as compared with other thermodynamic engines.

FIGURE 5

Thermal Efficiency of Power Plant

$$\eta_{th} = \frac{\text{Energy available for doing work}}{\text{Energy generated in the reactor}}$$



3. WEIGHT FRACTIONS

3.1 GENERAL

The gross take-off weight may be considered to consist of the following:

$$M_O = M_L + M'_E + M_S + M_P + M_T = \text{gross take-off weight}$$

where

$$M_L = \text{payload weight}$$

$$M'_E = \text{engine weight (reactor, pressure shell, pump and plumbing, and nozzle)}$$

$$M_P = \text{propellant weight}$$

$$M_T = \text{weight of tank and engine mount structure} \\ = f_T M_P$$

$$M_S = \text{weight of shielding} = f_S [M_O - M_P]$$

$$\text{Letting } M_E = M'_E + M_S$$

then

$$\frac{M_L}{M_O} = 1 - \frac{M_E}{M_O} - \frac{M_P}{M_O} [1 + f]$$

But

$$\frac{M_P}{M_O} = 1 - \frac{M_L + M_E + M_T}{M_O} = 1 - \exp \left[- \frac{\Delta V}{g_0 I_{sp}} + \frac{gt}{g_0 I_{sp}} \right]$$

so

$$\frac{M_L}{M_O} = [1 + f_T] \exp \left[- \frac{\Delta V}{g_0 I_{sp}} + \frac{gt}{g_0 I_{sp}} \right] - f_T - \frac{M_E}{M_O} \quad (3.1)$$

$$= [1 + f_T] \exp \left[- \frac{\Delta V + gt}{g_0 I_{sp}} \right] - f_T - 1.2 / [F_V / W_e] \quad (3.1a)$$

In the last term on the right side of this equation, it is assumed that

$$\frac{M_E}{M_O} \cong \frac{1.2}{F_V / W_E}; \text{ hence, } 1.2 \equiv F_V / W_O \equiv x F_a / W_O$$

where $x = F_v/F_a \cong 1.2$.

The factor 1.2 is used to empirically correct for the fact that thrust in vacuum, F_v , is greater than thrust in the atmosphere, F_a .^{*} A plot of this equation is given in Figure 6 for a particular choice of ΔV , assuming $\bar{g}t$ is negligible, for $f_T = M_T/M_P = 0.06$. This plot shows the trade-off for constant payload fraction between thrust-to-weight ratio of the engine and specific impulse when the rocket is used in a single stage (including boost). Thus, if the specific impulse is to be increased by a new engine concept, one must be certain that the thrust-to-weight ratio does not deteriorate to the extent that there is no net gain in payload fraction. (It should be noted that this curve may also be interpreted simply as a plot of the ratio of initial vehicle mass to engine mass, instead of F/W_E versus I_{sp} , and that the variation of F/W_E in the range 1 to 100 corresponds to a variation engine weight to initial vehicle weight of 1 to 0.01, respectively.) The curve shows a high incentive to achieve F/W_E ratios of 10 as a minimum (or ratio of engine to total mass of 0.10), and appreciable gain in payload occurs as the fraction is increased to 20; lower values of thrust-to-weight ratio impose very severe requirements on specific impulse. By way of contrast, Corliss shows for an electrical propulsion unit, the engine mass should equal the propellant mass (when structural weight is ignored); this corresponds to ratio of engine-to-total mass of 0.5.

3.2 TANK WEIGHT

Specific impulse varies directly as $\sqrt{T/(M.W.)}$ ⁺ where T is the absolute temperature and $(M.W.)$ is the molecular weight of the gas. Hence, a significant advantage in specific impulse stems from the use of hydrogen, the lowest molecular weight gas. Figure 7 shows I_{sp} versus temperature for several gases, including hydrogen; in the higher temperature range, dissociation of hydrogen becomes increasingly important so that I_{sp} increases faster than \sqrt{T} . To obtain specific impulses between 800 and 900 seconds, a temperature of the order of 4000°R is required; to achieve 2000 seconds, about 14,000°R is required; 3000 seconds requires greater than 20,000°R. Significantly greater temperatures are required if other gases were to be chosen as the propellant.

The effect of pressure level in the chamber is essentially negligible as it affects specific impulse provided the nozzle always discharges into vacuum; this would indicate that relatively low pressure levels might be desirable, except the size of the system will increase as pressure level

^{*} It is assumed that for boost F_a/W_0 must be greater than 1. If $F_v = F_a$, then $F_a/W_0 = 1.2$ at lift off.

⁺ See Appendix A-2 for derivation.

FIGURE 6

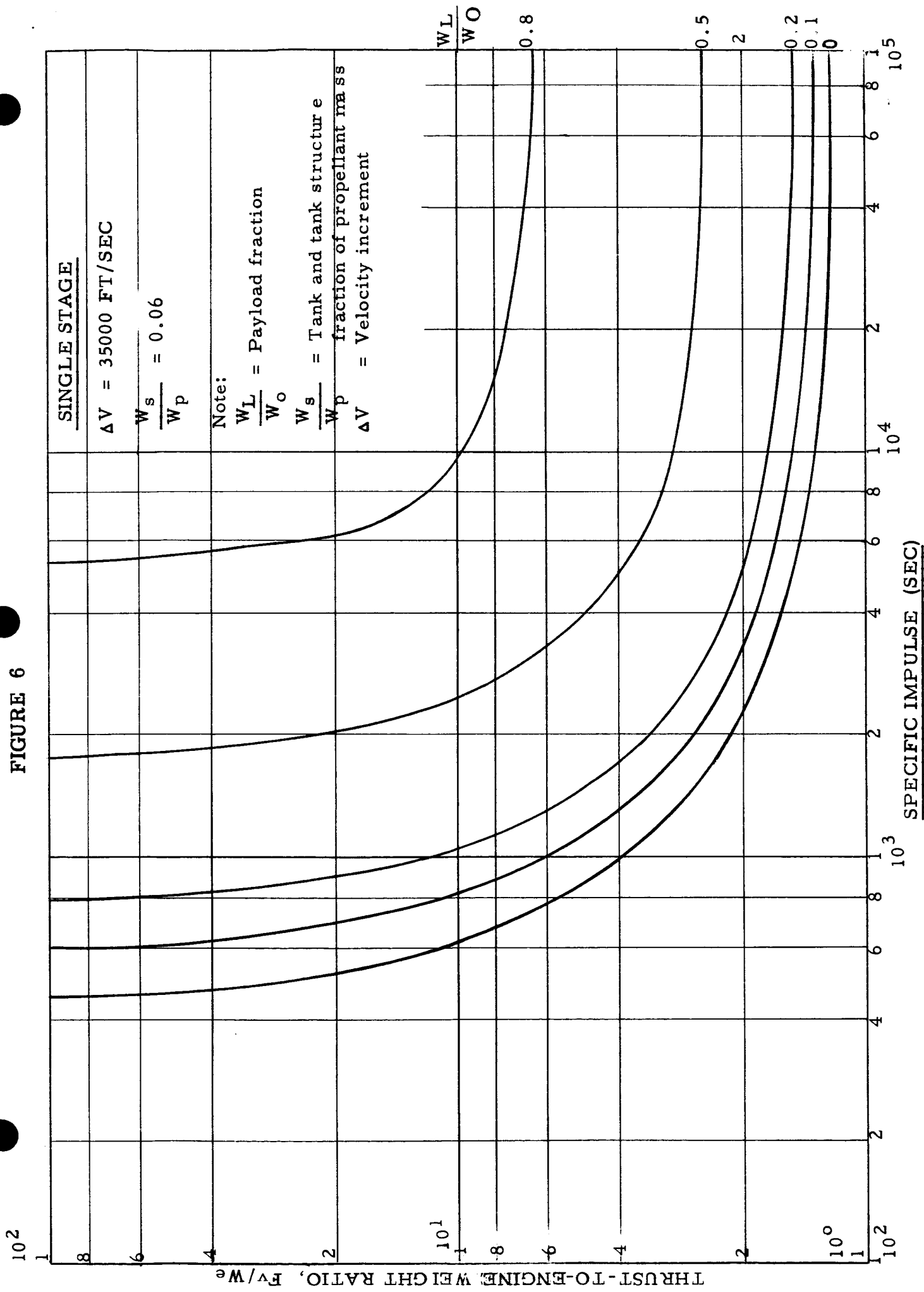
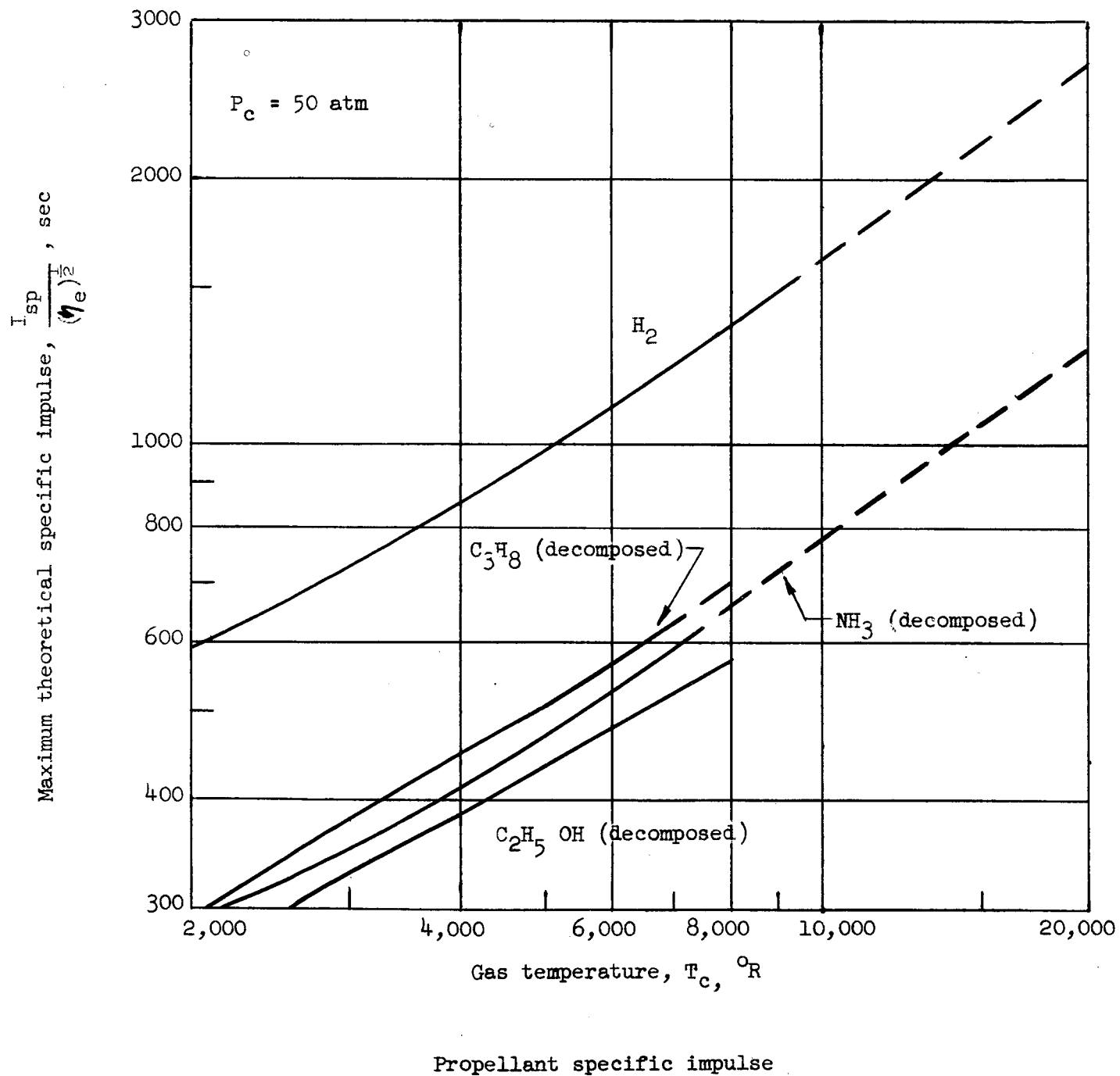


FIGURE 7



decreases. This is of concern primarily in the reactor where it is desirable to reduce the void area required to pass the gas through the fuel elements. Also it is desirable to minimize the areas of surfaces exposed to the hot propellant, since these are regeneratively cooled; this can be done only by reducing the size of the flow passages.

The impressive advantage of hydrogen in terms of its low molecular weight, consequently high specific impulse, is counterbalanced to some extent by its low density in the liquid state (4.3 lb per cu. ft.) and cryogenic temperatures required to remain liquid (-423°F). The tank weight relative to the propellant weight can be estimated by:

$$\text{Propellant weight: } W_P = \rho_P V_T = \rho_P \frac{\pi}{4} D^2 L \text{ for a cylindrical tank}$$

$$\text{Tank weight } W_T = \rho_T V_m = \rho_T \pi D t L \quad \text{for } t/D \ll 1.$$

then

$$\frac{W_T}{W_P} = \frac{\rho_T}{\rho_P} \cdot \frac{\pi D t L}{\frac{\pi}{4} D^2 L} = \frac{4 t}{D} \frac{\rho_T}{\rho_P} = 2 \frac{p}{S} \frac{\rho_T}{\rho_P}$$

The ratios of the tank-to-propellant densities divided by the room temperature yield strength of several possible tank materials are:

Tank Material	Be	Al (75S)	Ti-4Mn-4Al ^a	Low Alloy Steel ^a (Tricent)
Density Ratio $\frac{\rho_T}{\rho_P}$	26	40	66	112
Yield Strength S(psi)	37,000	20,000	140,000	240,000
$\frac{\rho_T}{S \rho_P} =$	0.7×10^{-3}	2.0×10^{-3}	0.47×10^{-3}	0.47×10^{-3}

^a "Aerospace Propulsion Data Book", G. E. Co. GED-4503

Assuming $\frac{\rho_T}{S \rho_P} = 0.5 \times 10^{-3}$, then $\frac{W_T}{W_P} = \frac{p}{1000}$.

If a reasonable tank-to-propellant weight fraction is 0.05, then the maximum allowable pressure is 50 psig. The corresponding thickness-to-diameter ratio for the titanium alloy is

$$t/D = p/2S = 50/2 \times 140,000 = 1.8 \times 10^{-4},$$

and the corresponding thickness for a 30 ft. diameter tank is ^{0.065}~~0.004~~ inches. The low alloy steel could also be used (if hydrogen embrittlement at low temperatures is not a problem), but at nearly half the thickness as the titanium alloy. If beryllium were chosen, the pressure must be reduced from 50 psi to 35 psi or the tank weight fraction increased from 0.05 to 0.07 in order to avoid exceeding its yield strength. Assuming the pressure is reduced, then

$$\left[\frac{t}{D} \right]_{\text{Be}} = \frac{35}{2 \times 37,000} = 0.47 \times 10^{-3}$$

and the thickness for a 30 ft. diameter tank would be 0.170 inches, over three times the thickness of the titanium alloy. The thicker tank material, other factors being equal, is preferable considering the thickness effect upon relative stiffness and protection from meteoroid penetration.

The boiling point of liquid hydrogen at one atmosphere pressure is 20°K (36°R); hence, containment at 35 to 50 psia at this temperature appears reasonable since it provides 20 to 35 psia head to the pump.

More detailed analysis and discussion of other factors involved in the problem of storing liquid hydrogen are given in NASA TN-D-587*; tank geometry, tank and supporting structure weight, meteoroid protection and heat input from nuclear and thermal sources are discussed.

3.3 WEIGHT OF GAMMA SHIELDING

The dose rate at the crew or payload position can be calculated by equation 3.2, below, if self absorption in the reactor, buildup in the shield,

* "Analysis of Liquid-Hydrogen Storage Problems for Unmanned Nuclear Powered Mars Vehicles", Brun, Livingood, Rosenberg, and Drier, Jan. 1962, (NASA TN D-587).

shield effect of structure and propellant, and scattering from vehicle components can be ignored; this simplification of the problem is suitable so long as one is primarily interested in establishing the trend in shield weight. (More detailed calculations must be made without these assumptions when a rigorous mission and system analysis is made.)

$$D = D_0 e^{-\Sigma X^*} \quad (3.2)$$

where

D_0 = Radiation dose rate in absence of shield

D = Radiation dose rate with shield

Σ = Linear attenuation coefficient for gamma radiation, cm^{-1}

X = Shield thickness, cm.

Solving for the shield thickness,

$$X = \frac{1}{\Sigma} \ln \frac{D_0}{D} \quad (3.2a)$$

and the weight, W_s , of the shield per unit frontal area is:

$$\frac{W_s}{A} = \rho X = \frac{\rho}{\Sigma} \ln \frac{D_0}{D} \quad (3.3)$$

where

A = frontal area of the shield.

The amount of attenuation required can be estimated from consideration of the amount of energy which must be expended in the reactor. The number of fissions required to generate 1 MW sec is:

$$1 \text{ MW sec} = 3.1 \times 10^{16} \text{ fissions.}$$

Approximately 7 mev of energy is emitted by the reactor in the form of prompt gammas and an equivalent amount is emitted by disintegrating fission products. In a typical case an additional 6 mev of gamma energy

* Glasstone, S., "Principles of Nuclear Engineering", D. Van Nostrand, Inc., New York, 1st . Ed., 1955.

is emitted as a result of (n, γ) reactions in reactor and component materials. Thus approximately 20 mev of energy is emitted in the form of gammas as a result of each fission event.

The energy flux intensity at any radius, r , is:

$$I = 3.1 \times 10^{16} \frac{\text{fissions}}{\text{MW-sec}} \times \frac{20 \text{ Mev}(\gamma)}{\text{fission}} \times \frac{1}{4\pi r^2 \text{ cm}^2}$$

$$= \frac{4.93 \times 10^{16}}{r^2} \frac{\text{Mev/cm}^2}{\text{MW sec}}$$

The conversion from energy flux intensity to dose rate depends upon the energy of the individual gamma photon, but a conversion factor of

$$1.7 \times 10^{-6} \frac{\text{rads/hr}}{\frac{\text{Mev}}{\text{cm}^2 \text{ sec}}}$$

is a close approximation over the entire range of gamma energies. Therefore, the unshielded dose rate is:

$$D_o = 1.7 \times 10^{-6} \left(\frac{4.93 \times 10^{16}}{r^2} \right)$$

$$= \frac{0.84 \times 10^{11}}{r^2} \frac{\text{rads}}{\text{MW hr}} \quad (3.4)$$

If we assume an allowable total dose of 25 Rad and the shield material is iron ($\rho = 7.8$ and $\Sigma = 0.265$ for gammas of 2-3 Mev),

$$\frac{W_s}{A} = \frac{7.8}{.265} \ln \frac{0.8 \times 10^{11} Q_{in} W_f}{25 r^2}$$

$$= \left(645 + 29.4 \ln \frac{Q_{in} W_f}{r^2} \right) \frac{\text{gm}}{\text{cm}^2} \quad (3.5)$$

where

Q_{in} = Reactor energy input per kg of system weight, MW hrs/kg

W_f = Weight in kg of system less propellant.

This shows that if the mission is such that the relative weight of the shielding is important in terms of the non-propellant mass fraction, the diameter of the reactor should be held to as small a value as possible.

Equation 3.5 for the shield weight is of the form:

$$X = C_1 \ln C_2 X$$

if we impose the condition that $W_S = f_S W_f$, where it is required that $f_S < 1$ for there to be any weight allowance for the tankage, engine, structure, and payload. The solution of this equation for shield diameters of 1, 5, and 10 ft. at a separation distance of $r = 100 \text{ ft.} = 3050 \text{ cm.}$ between crew and reactor, is given in Figure 8 and Table 6. In this case the shield weight is given by:

$$W_S \text{ (kg)} = \frac{A}{1000} \left[173 + 29.4 \ln (Q_{in} W_f) \right]$$

and

$$W_S = f_S W_f; f_S = 1, 0.5, 0.25.$$

The numerical results show that the shield weight is determined primarily by the cross-sectional area of the shield, and the variation of W_f (by a factor of 4) affects the shield weight by only 10%. Similarly, Q_{in} may be varied by a factor of 100 with less than 50% affect upon the shield weight. Hence, Table 6 may be interpreted as giving a good first approximation of gamma shielding requirements for interplanetary missions included in Tables 5A and 5B.

3.4 MINIMUM DRY WEIGHT INCLUDING SHIELD

Therefore, an indication of the minimum over-all weight of the rocket is obtained. Larger weights are tolerable; smaller weights may not provide sufficient shielding, depending upon the results of more accurate analyses (these estimates are probably within a factor of 2). As a rule of thumb for further assessment, to hold the shield weight to 25% of the non-propellant mass of the vehicle for a total reactor energy

FIGURE 8

Shadow Shield Weight per Unit Frontal Area

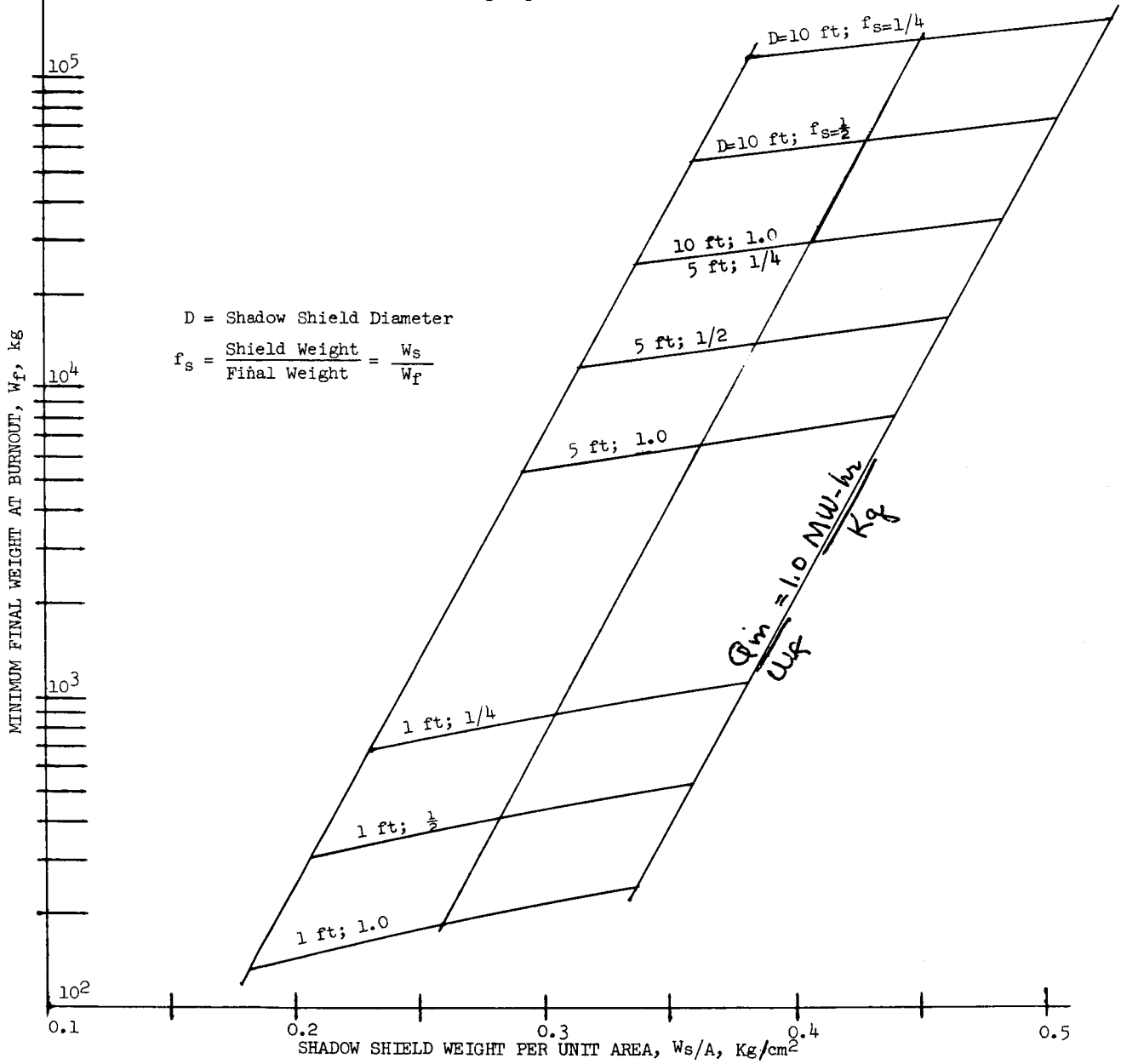


TABLE 6

Shield Weight Requirements (Kg)
(for Gamma Radiation, Using Iron)

Diameter of Shield (ft) =		1		5		10	
Area of Shield (cm ²) =		730		18200		73000	
		$\frac{W_s}{W_f}$	$\frac{Q_{in}}{W_f}$	$\frac{W_s}{W_f}$	$\frac{Q_{in}}{W_f}$	$\frac{W_s}{W_f}$	$\frac{Q_{in}}{W_f}$
$\frac{W_s}{W_f} = 1.0$	$\frac{Q_{in}}{W_f} = \begin{cases} 1.0 & \text{MW hr} \\ 0.1 & \text{"Kg} \\ 0.01 & \text{"} \end{cases}$	245	245	8000	8000	35200	35200
		189	189	6620	6620	29800	29800
		134	134	5310	5310	24400	24400
$\frac{W_s}{W_f} = 0.5$	$\frac{Q_{in}}{W_f} = \begin{cases} 1.0 & \text{MW hr} \\ 0.1 & \text{"Kg} \\ 0.01 & \text{"} \end{cases}$	262	523	8350	16700	36800	73500
		207	413	7000	14000	31300	62600
		150	300	5700	11400	26200	52300
$\frac{W_s}{W_f} = 0.25$	$\frac{Q_{in}}{W_f} = \begin{cases} 1.0 & \text{MW hr} \\ 0.1 & \text{"Kg} \\ 0.01 & \text{"} \end{cases}$	278	1110	8800	35200	38500	154000
		223	892	7450	29800	33000	132000
		167	668	6100	24400	27800	111200

Note: W_s = Shield weight, Kg, required to limit crew dose to 25 rad at a distance of 100 ft. from reactor with a total energy increment equal to Q_{in} .

W_f = Minimum non-propellant weight of vehicle, Kg, required to carry the necessary shield.

increment of 1 MW-hr/Kg (suitable for surface-to-surface missions from Earth to the planets and return), the mass of the vehicle after all the propellant is used should be:

TABLE 7

Minimum Vehicle Mass for Interplanetary Missions

<u>Shield Diameter</u>	<u>Minimum Vehicle Mass^a (less Propellant)</u>
1 ft.	1.2 tons
5 ft.	39 tons
10 ft.	170 tons

-
- a. For shield weight 25% of final weight and 1 MW-hr/Kg reactor energy increment. Shield weight estimate probably accurate with a factor of 2. Missions requiring only 0.01 MW-hr/Kg need only about 2/3 these values.

Greater vehicle masses will result in a smaller fraction required for shielding.

3.5 SUMMARY OF WEIGHT FRACTIONS

The discussion to this point has reviewed some of the basic factors leading to preliminary estimates of the range of weight fractions which must be expected for various components. At the same time, the conditions which must be imposed upon the reactor have been roughly identified. This is summarized below (single stage rocket):

<u>Component</u>	<u>Weight Fraction</u>	<u>Comments</u>
Propellant	$0.66 < M_P < 0.80$	Allows EARTH escape with $700 < I_{sp} < 1000$ sec, or hydrogen temperature $3000 < T_c < 5000$ R.
Tank	$0.04 < M_T < 0.12$	Lower value suitable for simple pressure containment at liquid hydrogen temperature; higher value reflects addition of meteoroid protection and thermal insulation.
Engine	$0.05 < M_E < 0.10$	Arbitrary value unless boost from EARTH surface required. Then, highly dependent upon thermal, nuclear, and mechanical characteristics of reactor. Small sizes nuclearly controlled; large sizes controlled by flow area required.
Shield	$0.05 < M_S < 0.08$	Fixes minimum vehicle size depending upon reactor diameter. Manned vehicles assumed. Fraction can be decreased by increasing vehicle size.
Total non-propellant	$0.14 < M_f < 0.30$	
Payload	$0.04 < M_L < 0.20$	

In view of the fact that shield weight fixes the minimum vehicle mass, the minimum lift-off weight is fixed once the propellant fraction is fixed; likewise, if the boost from the pod or if 1g initial acceleration is desired, the minimum thrust is fixed. These numbers are summarized below:

TABLE 8

Minimum Initial Vehicle Weight and Thrust

<u>Shield Diameter</u>	<u>Vehicle Weight at Burnout</u>	<u>Initial Weight^a</u>	<u>Thrust^b</u>
1 ft	1 ton	$3 < M_0 < 5$ tons	10,000 lb
5 ft	35 tons	$105 < M_0 < 175$	350,000 lb
10 ft	150 tons	$450 < M_0 < 750$	1,500,000 lb

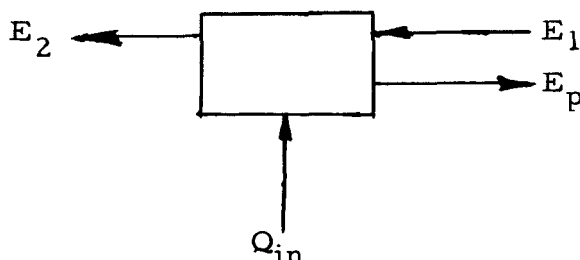
a. For propellant fraction $0.66 < M_p < 0.80$.

b. For 1g minimum acceleration; 5g maximum acceleration at burnout.

APPENDIX A-1

ENERGY BALANCE AND PROPULSIVE EFFICIENCY

In some maneuvers, the energy output from the reactor can be less than the vehicles energy increment. A simple energy balance shows that the kinetic energy of the ejected propellant supplies the difference. Consider the following:



Let E_1 = initial kinetic energy of vehicle and propellant
 $= \frac{1}{2} M_1 V_1^2 = \frac{1}{2} (M_2 + \Delta M) V_1^2$

E_2 = final kinetic energy of vehicle
 $= \frac{1}{2} M_2 V_2^2$

Q_{in} = reactor energy added to the system

E_p = kinetic energy contained in the ejected propellant
 $= \frac{1}{2} (\Delta M) (\bar{V} - V_j)^2$

where \bar{V} = the properly averaged velocity of the vehicle during the pulse from V_1 to V_2 .

V_j = the velocity of the jet with respect to the vehicle (assumed constant)

ΔE = energy increment imparted to the vehicle
 $= \frac{1}{2} M_2 (V_2^2 - V_1^2)$

Then, the energy balance yields:

$$Q_{in} + E_1 = E_2 + E_p$$

or

$$Q_{in} + 1/2 (M_2 + \Delta M) V_1^2 = 1/2 M_2 V_2^2 + 1/2 \Delta M (\bar{V} - V_j)^2$$

$$\begin{aligned} Q_{in} + 1/2 \Delta M V_1^2 &= 1/2 M_2 (V_2^2 - V_1^2) + 1/2 \Delta M (\bar{V} - V_j)^2 \\ &= \Delta E + 1/2 \Delta M (\bar{V} - V_j)^2 \end{aligned}$$

The total input is $Q_{in} + 1/2 \Delta M V_1^2$ and the useful output is ΔE . Therefore

$$\eta_p = \frac{\text{useful output}}{\text{total output}} = \frac{\Delta E}{Q_{in} + 1/2 \Delta M V_1^2}$$

If the initial velocity, V_1 , is zero, then Q_{in} is always greater than ΔE . On the other hand, the maneuver involving escape from low earth orbit with a nuclear rocket represents a case in which Q_{in} is less than ΔE . As an illustration, consider the two steps involved in launching from the surface of the earth to orbit at low altitude, followed by the escape pulse. From Table 1, the well depth for earth is 6.25×10^7 joules/Kg and the escape velocity is 36,700 ft/sec. From equation 2.9, the orbit velocity is 26,000 ft/sec; also, from equation 2.10a and 2.11b the energy increment required for orbit is equal to the energy requirement for escape from orbit (for an orbit at low altitude, $n = 1$). Then the data tabulated below applies when the specific impulse is 850 sec.

TABLE A-1

Earth Escape Vehicle and Reactor Energy Increments							
Step	Vehicle Energy Increment (joules/Kg)		Velocity Increment (ft/sec)	Mass Ratio (Isp = 850 sec)		Reactor Energy Increment (joules/Kg)	
	$\frac{\Delta E}{M_n}$	$\frac{\Delta E}{M_f}$		$\frac{M_{n-1}}{M_n}$	$\frac{M_n}{M_f}$	$\frac{Q_{in}}{M_n}$	$\frac{Q_{in}}{M_f}$
1	3.125×10^7	4.62×10^7	26000	2.582	1.478	5.49×10^7	8.11×10^7
2	3.125×10^7	3.12×10^7	10700	1.478	1.000	1.66×10^7	1.66×10^7

$$\frac{M_o}{M_f} = 3.82$$

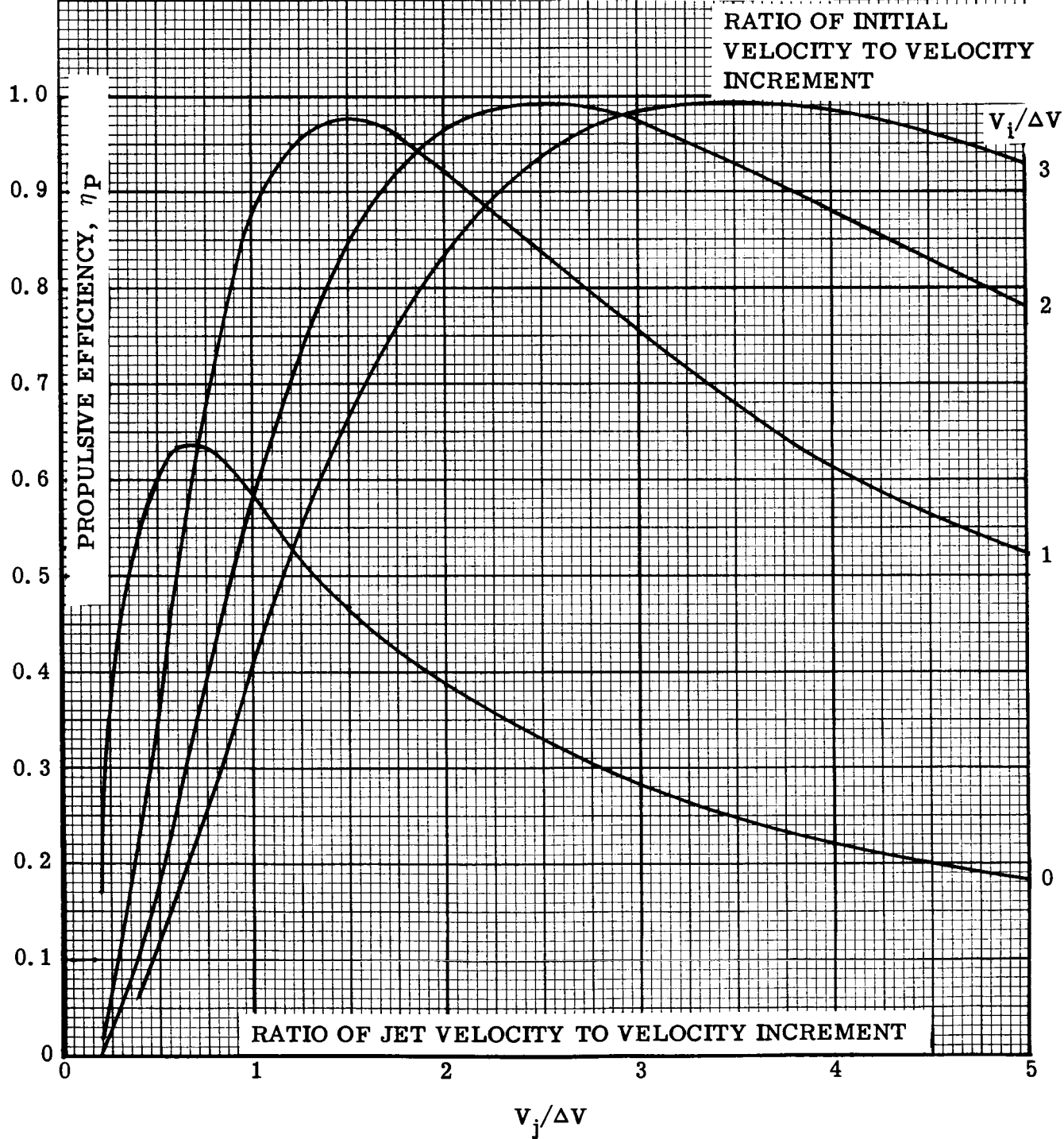
$$\sum \frac{Q_{in}}{M_f} = 9.77 \times 10^7$$

In step 2 of the above table, the reactor energy increment required is nearly half the energy imparted to the vehicle. If both steps 1 and 2 were to be done with a single stage, this factor would be of no concern; however, if the first step were done with a chemical rocket and the second with a nuclear rocket, then the influence of initial velocity upon reactor energy requirement is significant.

Propulsive efficiency is plotted in Figure A-1, given in terms of the parameters $V_j/\Delta V$ and $V_1/\Delta V$ which are the ratios of jet velocity to velocity increment and initial velocity to velocity increment, respectively. This plot is primarily of interest for reference purposes, since one has no control, generally, over the two parameters involved once a mission requirement is specified.

FIG. A-1
PROPULSIVE EFFICIENCY

$$\eta_P = \frac{\Delta E}{Q_{in.} + \frac{1}{2} \Delta M V_1^2}$$



APPENDIX A-2

RELATIVE ENERGY REQUIREMENTS FOR
IMPULSE VERSUS CONTINUOUS THRUST MODES

a) Instantaneous impulse

Equation 2.9 shows:

$$V_e = \sqrt{2} V_o$$

or

$$\Delta V = V_o (\sqrt{2} - 1)$$

This implies the entire energy increment is applied instantaneously to the vehicle at the start of the maneuver and the vehicle coasts out to escape.

b) Continuous thrust

In this case:

$$\Delta V = V_o$$

This is derived considering:

$$E_t = \frac{\gamma M M_p}{2 r} \quad \text{for a vehicle in orbit.}$$

For no change in the mass of the vehicle, M , during thrust application (as for an electromagnetic propulsion device) ;

$$dE_t = \frac{\gamma M M_p}{2 r^2} dr = F r d\theta$$

$$\text{or} \quad dr = \frac{2 F R}{W} \left(\frac{r}{R} \right)^3 d\theta$$

where

$$\gamma M_p = g_o R^2$$

and

$$W = g_o M$$

But

$$V_s = r \frac{d\theta}{dt}$$

So

$$dt = \frac{r}{V_S} d\theta = \sqrt{\frac{R}{g_0}} \left(\frac{r}{R}\right)^{3/2} d\theta$$

where

$$V_S = \sqrt{\frac{\gamma M_p}{r}} = R \sqrt{\frac{g_0}{r}}$$

therefore

$$t = \sqrt{\frac{R}{g_0}} \frac{W}{2F} \int_{r_0/R}^{r/R} \left(\frac{r}{R}\right)^{-3/2} d\left(\frac{r}{R}\right)$$

So

$$\begin{aligned} t &= \sqrt{\frac{R}{g_0}} \frac{W}{F} \left[\sqrt{\frac{R}{r_0}} - \sqrt{\frac{R}{r}} \right] \\ &= \frac{WR}{F\sqrt{g_0 r_0}} \left[1 - \frac{1}{\sqrt{n}} \right] \end{aligned}$$

where

$$n = \frac{r}{r_0}$$

or

$$Ft = \frac{WR}{\sqrt{g_0 r_0}} \left[1 - \frac{1}{\sqrt{n}} \right] = \frac{W}{g_0} \Delta V$$

therefore

$$\begin{aligned} \Delta V &= R \sqrt{\frac{g_0}{r_0}} \left[1 - \frac{1}{\sqrt{n}} \right] \\ &= V_0 \left[1 - \frac{1}{\sqrt{n}} \right] \end{aligned}$$

For escape $n \rightarrow \infty$ and $\Delta V = V_0$.

APPENDIX A- 3

JET VELOCITY AS A FUNCTION OF PROPELLANT MATERIAL
AND TEMPERATURE

Since the jet velocity is achieved thermodynamically in the nozzle under the conditions of no heat addition or work extraction, the energy balance between the chamber and the nozzle exhaust shows:

$$\mu_1 + \frac{p_1 v_1}{J} + \frac{V_1^2}{2 g_o J} = \mu_2 + \frac{p_2 v_2}{J} + \frac{V_2^2}{2 g_o J}$$

or

$$h_{s1} + \frac{V_1^2}{2 g_o J} = h_{s2} + \frac{V_2^2}{2 g_o J}$$

or

$$h_{T1} = h_{s2} + \frac{V_2^2}{2 g_o J}$$

where

$$h_s = \mu + \frac{pv}{J}$$

$$h_T = h_s + \frac{V^2}{2 g_o J}$$

So

$$\begin{aligned} V_2 &= \sqrt{2 g_o J (h_{T1} - h_{s2})} \\ &= \sqrt{2 g_o J C_p (T_{T1} - T_{s2})} \\ &= \sqrt{2 g_o J C_p T_{T1} \left(1 - \frac{T_{s2}}{T_{T1}}\right)} \\ &= \sqrt{2 g_o J \frac{\gamma}{\gamma - 1} \frac{RT_{T1}}{J(\text{M. W.})} \left[1 - \left(\frac{p_{s2}}{p_{T1}}\right)^{\frac{\gamma - 1}{\gamma}}\right]}, \text{ for } C_p = \frac{\gamma}{\gamma - 1} \cdot \frac{R}{J(\text{M. W.})} \end{aligned}$$

$$= \sqrt{2 g_0 \frac{\gamma}{\gamma - 1} \frac{RT_T}{(\text{M.W.})}} \text{ as } p_{s2} \rightarrow 0$$

and

$$I_{sp} = \frac{V}{g_0} = \sqrt{\frac{2\gamma R}{(\gamma - 1) g_0}} \sqrt{\frac{T}{(\text{M.W.})}}$$

where

γ = ratio of specific heats

R = universal gas constant

(M.W.) = molecular weight of gas